Customer Value Modelling: Model synthesis and extension proposals

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INTRODUCTION

Relationship Marketing and Direct Marketing

The development of a "customer oriented" interactive and database marketing brings to forefront, in most contexts, customer value and customer portfolio assessment methods. These methods have either been fine tuned in direct marketing and in catalogue sales or developed in certain sales force effort allocation models. Improvement possibilities for these methods are still numerous but the road towards such progress passes through a systematisation and unification effort.

Profitability of the customer relationship

From a customer attraction and retention perspective, a profitable customer is a customer whose incomes, generated during the commercial relationship, exceed with an acceptable amount costs supported to attract, satisfy and keep him. This amount is called customer value or lifetime value of the customer.

Need for customer value calculations

Customer value calculations help enterprises solve various fundamental problems like budgeting customer acquisition expenses, selection of recruiting media (the LTV is generally different according to the medias used), or of types of offer or distributing efforts between prospecting and preserving customers. Well conducted, LTV analysis can also help build a competitive strategic advantage (Jackson, 1996).

Dichotomies of the customer's relationship behaviours

The modelling of customer value depends on the context and on the customer's relationship behaviour (whether the situation is contractual or not). A brief analysis shows that modelling efforts are based on the distinction between customer retention and migration.

Research objective

This study of existing models shows also that, in a first set of contributions, the mathematical developments are algebraic and focus on retention models while, in the another set, they are based on matrix approaches and applied mainly to the migration model.

The need to desynchronise financial flows (expenses and gains) in the customer relationship, is also put forward by these contributions.

Based upon results emerging from a broad examination of these models, this article guides towards systematisation and unification. It finds elements that are common and identifies those that are not. It adds components that seem absent from the other approaches. In this way, a set of formulas that complete existing solutions is developed and adapted.

Research Framework

These, existing or new, formulas are grouped in a progressive customer value calculation framework. It is presented schematically in the figure 1 and in detail in the table of the appendix 1.

Figure 1. – Modelling customer value, a research framework

The algebraic approach and the matrix approach are treated apart. Formulations progressively integrate transactional flows expressing probabilities to pass orders, financial flows composed of gains and expenses and customer and prospect value optimisation procedures based upon long-term calculations.

The use of certain representation artefacts (migration trees and diagrams, transition probability matrixes) facilitates the development of some recursive algebraic formulas for LTV calculations.

Several additional aspects concerning dynamics of customer migration, like right and left censored migration processes or mechanics of purging the customer list, are treated explicitly.

Retention, migration and mathematical approaches of customer value

Some works on customer value

Dwyer (1989) presents examples of LTV calculations drawn from the two major contexts that were distinguished by Jackson(1985) in the industrial buying environment: buyers of type "always a share" and buyers of type "lost for good".

On this occasion Dwyer (1989) reveals the more general character of this typology, extends it to the consumer market, and shows that the "always a

share" behaviour, that he associates to the "migration model", is generally representative for catalogue buying; while the "lost for good" behaviour, that he associates to the "retention model", is representative for financial services, press subscriptions, etc.

Algebraic approaches

Berger and Nasr (1998) use this distinction between customer retention and migration and present mathematical LTV calculation formulas for four situations implying retention models and for one context requiring a migration model.

Blattberg and Deighton (1996) build a model that computes "customer equity" and finds the optimal balance between customer acquisition (investments to convert prospects into customers) and customer retention (investments to convince active customers to continue to buy) expenses. This model uses an infinite time horizon (long term) and LTV formulas are therefore simple, easy to use and avoid all summing operations that limited time horizon (life time) models suppose. The concept of customer equity is further developed in Blattberg, Getz and Thomas (2000).

Matrix approaches

Building upon these works of Berger and Nasr (1998) and of Blattberg and Deighton (1996), Pfeifer and Carraway (2000) apply Markov chains to customer relationship modelling and LTV calculations. They insist on the flexibility of models based on Markov chains and show that all situations modelled by the previous authors can be solved with Markov chain models. While the previous contributions are limited to recency based customer value models, these authors illustrate the link between Markov chains and the RFM (Recency, Frequency and Monetary) modelling framework that dominates industry practices in Direct Marketing and Catalogue sales.

Some developments that are more specific to these sectors can also be found in Birtran and Mondschein (1996, 1997).

Comparison of works

A comparative analysis of these contributions brings several points to the forefront, which underline the need of a systematisation and unification effort:

The nature and mechanics of the customer's individual response

Dichotomies of the customer relationship behaviour

The two types of temporal behaviour of customers in a relation with a company (that have been called differently according to authors: "lost for good"/"always has share" for Jackson, "retention model"/"migration model" for Dwyer or " contractual" / "non-contractual" for Reinartz and Kumar), display different migration patterns.

Particularities of the retention model

The retention model considers that a person or a company remains a customer as long as she generates transactions. This means that if at some given moment the customer doesn't renew his contract or doesn't generate any transaction he can be considered as "lost for good" or as an "ex-customer". It also means that if an "ex-customer" buys again he is considered as a new customer and one deals with an acquisition rather than a customer retention issue. The evaluation of customer potential in a relation of this kind will only take into account the customer's probability to remain active from one period to an another or what Blattberg and al. call the customer's survival probability. The customer lifetime corresponds to the number of successive periods during which the customer is and remains active.

Particularities of the migration model

The migration model considers that customers can re-appear (turn up again) after some periods during which they didn't make transactions and traces their

probability to "reactivate". The evaluation of the customer's potential in such a relationship relies on the joint probabilities of remaining and becoming again an active customer after a fixed number of periods or in other words on the " survival " and "reactivation " probabilities. The lifetime of the customer corresponds to the number of successive periods during which the customer, according to the company's estimations, either remains active or reactivates. For a customer to preserve this status after several periods of inactivity his probability to reactivate, as estimated by the company, must to be superior to the prospects' response probability. Alternatively the customer is ticked off the customer database and he becomes an ex-customer or a "purge" if one adopts a certain terminology used by the direct marketing profession.

Distinction between transaction- and financial flows

In order to encourage a progressive assessment of customers' potential and to facilitate the resolution of problems with increasing complexity it is convenient to distinguish transactional and financial flows that were generated by customers during the number of periods considered. The first will help estimate the number of transactions incurred by a customer during the relationship with the company and the others will facilitate profitability calculations for the customer relationship.

Probability to generate transactions and transaction potential

The status of a customer is given by an initial transaction. In order to asses transactional flows it is appropriate to count the transactions generated during several periods or business cycles by a number of initial transactions t_0 or customers. Let's note t_j the number of transactions generated by t_0 customers or initial transactions after j periods. The customer's response probability after j periods is given by the number of transactions generated after j periods divided by the number of initial transactions or t_j when $t_0 = 1$. The transaction potential of a customer will then be given by the expected number of transactions generated during the lifetime of a customer. This corresponds at each period to the sum of probabilities to generate a transaction: $T_j =$

The Retention Model

Introduction to the Retention model

The "lost for good" type of behaviour implies that the customer remains active until the moment he leaves the company. Simplified, this means that at each business cycle those who remain customers are going to generate a transaction.

The general case (variable retention probability with time)

The management of subscribers to a consumer magazine represents this situation well and raises a generic problem of customer retention and attrition. The customer potential and the customer value come from individuals with whom the relation persists. In an example drawn from the consumer magazine subscription business, Dwyer presents a situation in which the probability to remain a customer by renewing the subscription increases slightly with time and becomes stable after some periods.

Numeric example

Dwyer gives a numeric example that is repeated by many of the other authors mentioned before. We summarize and adapt this example in order to reveal essential mechanisms of the retention model and to allow for evaluation of similar situations in other industries by decision calculus. It deals with a company in the magazine subscription business who evaluates the transactional potential and the value of its customers for an estimated lifetime of five periods. The margin by retained customer is 40$ of which 20$ are incomes from subscriptions and 20$ are advertising revenues. The marketing costs by retained customer are 28$ and the discount rate is 20%.

Table 1 – The magazine subscriptions case (growing retention rate)

Formalisation of calculations

The survival probability at each period j is , the product of probabilities to remain a customer during all those periods. It indicates the expected number of transactions per customer on each period (tj).

The cumulated expected number of transactions per customer during the analysed periods is the sum of survival probabilities: Tj = . In order to facilitate

customer value calculation we introduce the notion of discounted transactions [1] . The expected and discounted number of transactions during a customer lifetime is given by the discounted sum of his survival probabilities: Tja = , where a is the discount rate and j is the lifetime of a customer expressed number of periods. In the example, the expected number of transactions during the lifetime of a customer is 3.12 (see Figure 1.) and its discounted version is 2,43.

The expected value of the customers (Vj) is the profit of a transaction multiplied by the discounted expected number of transactions: .

The case where retention probabilities remain constant

Particularities of models with constant retention rate

In mature market situations one can suppose that for a constant marketing effort purchase probabilities remain constant from one period to another. Under these circumstances formulas are simplified and make it easier to estimate long-term evolutions. As a result survival probabilities can be expressed as powers of the constant retention probability and no more as a product of variable probabilities.

Numeric Example

Dwyer's example adapted to a situation with constant retention probability leads to the following calculations (see table 2).

Table 2 – The magazine subscriptions case (constant retention rate)

Formalisation of calculations

The survival probability after j periods is , the probability to remain customer to the power of the number of periods.

The sum of survival probabilities gives the transaction potential of a customer which is here = 3,36. The discounted transaction potential is then that is =2,6.

The net present value of a customer results from the profit of a transaction multiplied by the discounted transaction potential: that is (40$ - 28$) 2,6 = 31,2 $

"Long-term" customer value

Use of the long-term analysis

When purchase probabilities remain constant from one period to the other, it is convenient to compute the long-term present value of a customer. The long term indicates a hypothetical situation where the customer's lifetime would be endless. Computing the long-term transaction potential and customer value is not of great interest in itself. It helps optimise marketing efforts: avoiding to spend too much in customer retention programs with no budget constraints or finding the optimum retention budget when budget constraints exist.

Transactional Potential and long-term customer value

It can be shown that the long-term transaction potential is $1/(1-p)$. On the long term, the discounted transaction potential will then be = $(1+a)/(1+a-p)$.

For example if the (constant) retention probability is 0.8 then the long-term transaction potential of a customer is $1/(1-0.8) = 5$. This means that if the lifetime of a customer lasts forever that customer would generate 5 transactions.

The discounted transaction potential is $(1+0.2)/(1+0.2-0.8)=3$ and the customer's present value will be $(m-c) (1+a)/(1+a-p)$ or $(40\$-28\$)*3 = 36\$$.

Customer's Long-term and Lifetime values

If one remembers that the discounted transaction potential of a customer was

2,6 transactions and the customer's present value was 31,2$, it becomes clear that these long-term measures must be seen as limits towards which tend the cumulated transactions and the customer value.

These limits offer a fast computing instrument that avoids fastidious summations and produces some indicative values of the transaction potential and of the customer.

Optimisation of the long-term present customer value

Retention probability: as a function of customer oriented expenses

Up to this point the calculations have used given retention rates. It is yet justified to consider that the company can influence retention rates through marketing actions oriented towards its customers. This supposes to express the customer retention probability as a function of the marketing effort (budget) directed towards the customers during each period. The retention probability expressed as function of the retention budget increases with a growth rate that decreases progressively towards the end when the retention probability reaches its ceiling (see figure 1). A simple exponential function like pr = ceiling * (1-exp(b * R)) can be estimated by decision calculus using subjective estimations from mangers. Numeric example

Finding the budget that maximizes the customer's present value

The optimum retention budget for a customer R can be found by maximising the present long-term customer value, that is max(m-R/pr) (1+a)/(1+a-pr) where pr = f(R)

Figure 2 - Maximization of the long-term customer Value (without the acquisition stage)

Numeric Example

In the example with a 23$ retention budget R by customer the company achieves a 0.8 retention rate, this corresponds to 28$ by retained customer and gives a long-term present value of the customer of (40-28)*(1+0.2) / (1+0.2-0.8)=36$. If the response to the retention budget is expressed by the function pr =0.81(1-exp(-0.16*R) one can see that this result is not an optimum one.

In such a situation, where the retention rate is limited upwards to 0.81 and marks an accelerated growth (coefficient of elasticity 0.16) with an effort of only 7$ per customer a retention rate of 0.785 is obtained. This results in 8,9$ per retained customer. The long-term value becomes then (40-8,9)*(1+0.2) / (1+0.2-0.785)=90$ which is optimum.

Finding the balance between customer acquisition and retention

Acquisition and Retention Probabilities as functions of prospect/customer oriented expenses

In order to have the complete picture of the customer relationship profitability, one cannot ignore the customer acquisition stage that transforms prospects into customers. Blattberg and Deighton (1996) call the measure of customer profitability "customer equity". By putting the problem in a prospect's[2] perspective they suggest a model that makes it possible to optimise customer acquisition and retention.

and = p a [m-c a +(m-c r) *p r /(1+a-p r)]

Following the reasoning previously applied to retention costs, customer acquisition costs ca can be expressed as expenses by prospect divided by the acquisition probability pa, which itself is a function of these expenses. Numeric example.

Numeric Example

Insuring customer relationships profitability reduces to successive optimisation of the customer acquisition and retention value.

Max[p a m - A + p a (m p r - R)/(1 + a - p r)]

where a = f(A) p and r = f(R)

The optimisation procedure, that has been adapted from Blattberg and Deighton, finds first prospect expenses that maximize the acquisition value (p a m - A) of a customer and then uses the resulting acquisition rate (p a ') in order to compute the retention value (p a ' (m p r - R)/(1+a-p r)). The maximum customer value is the sum of the optimum acquisition and retention values calculated in this way. Due to its simplicity and conciseness this model is well adapted to decision calculus.

Numeric Example

In order to set the Blattberg and Deighton problem in the dual perspective of customer acquisition and retention, it is first necessary to express the acquisition probability as a function of acquisition expenses by prospect, for example pa = 0.4 (1-exp(-0.1 * A)). The ceiling acquisition rate (probability) and its elasticity should be lower then their corresponding parameters in the retention rate function. The optimised values can be found graphically (see in Figure 2).

With a 5$ budget per prospect a 0.16 response probability (acquisition rate) is reached. It gives the best acquisition value for a prospect 0.16*40-5 = 1,53 $ and can be read on the negative scale in figure 2. This acquisition rate enters the calculation of the retention value which becomes maximum when retention expenses by customer are 7$ and result in a retention rate of 0.78. In this way, the maximum retention value that can be achieved per prospect is 9,62$. The total value of a prospect is the sum of his acquisition and retention values 1,53$+9,62$= 11,16$. The customer value can be easily derived here from by dividing the prospect value by the acquisition rate (probability) 11,16$/0.16 = 68,32 $. The result is coherent with the 90$ customer value obtained when

acquisition costs were ignored.

Figure 3 - Maximization of the customer value (with the acquisition stage)

Desynchronisation of the financial flows in the retention model

Synchronized flows

In their survey of retention models Berger and Nasr (1998) pay a special attention to the distinction between the moments when input (gains) and output (expenses) flows intervene. In the previous examples we considered that gains (m) and promotional expenses (c) intervened at the same moment during a business cycle where from the formula .

Desynchronisation of flows and commercial cycle amplitudes

A more attentive analysis of the alternation of gains and expenses of a company during the life of a customer, indicates the following timing: If acquisition costs that occur in the previous cycle are ignored, the customer brings gains at the beginning of the cycle and the company makes its promotional expenses at the middle of cycle in order to assure repeat purchase. Cycle lengths can be equal, lower or superior to one year. There can be different cycles for gains and for expenses.

Formalisation and numeric example

The following formula generalizes and regroups several formulas suggested by Berger and Nasr (1998), in order to deal with various situations:

$V_j = -$

where cm and cc are the number of cycles per year for gains and for expenses.

Table 3 - Cases that illustrate commercial cycle particularities and the desynchronisation between the financial input (gains) and output (expenses) flows in different industries

Berger et Nasr (1998) give detailed calculations for the three cases summarised here and for two other cases presenting situations where profits per retained customer aren't constant but variable with time (case 3) and situations where financial flows aren't discrete but continuous (case 4).

The Migration Model

Mechanics of retention/reactivation - a series of artefacts

The "always a share" type of behaviour supposes that the customer doesn't have to be active during all the analysed period and that he can reactivate with a probability that increases with recency of the last transaction[3].

In order to get a better understanding and use of retention/reactivation mechanisms and of stochastic processes that characterize this kind of behaviour, several artefacts like decision trees, migration trees, diagrams and matrixes of transition probabilities are used. They facilitate the development or adaptation of a series of algebraic and matrix formulas.

Purchase profile (pattern) and the decision tree

With time customers have purchase patterns that alternate active and inactive periods. A prolonged inactivity can lead to elimination from the customer list. The progressive shaping of purchase profiles is visible in figure 4:

Figure 4 - Stochastic decision tree and purchase patterns

By regrouping customer purchase patterns at each stage according to recency criteria, by first separating the active customers from those that are not active and by segmenting afterwards the inactive ones according to the recency of their last purchase a customer migration tree is obtained. It is a key representation form for understanding transition mechanisms in this model.

The migration tree and the recurrence of the transition process

The migration tree shows particularly well (see Figure 5.) the recurrent character of the transition process to which the customer is submitted. This facilitates the development of powerful and flexible algebraic formulas for computing the transactional and financial flows associated to the migration model.

Figure 5 - Customer migration tree

Formulas for a migration process censored in relation to the recency

The expected number of transactions generated by a customer after j periods is the sum of reactivation probabilities of transactions initiated by the same customer in the previous periods. It is a recurrent expression that has as a starting point the first transaction, the one that turns a prospect into a customer:

This formulation is more general and more concise than the formulas given by Berger and Nasr (1998, eq. 10. p.25) and Dwyer (1997, p. 12). It integrates a recency limit, noted R that enables right censoring of the migration process and makes this expression and the other ones that have been derived there from comparable to the matrix formulations suggested by Pfeifer and Caraway (2000).

Censorship according to recency is largely used by the industry. When a segment or a customer crosses this recency limit, his response probability becomes lower or equal to the prospects' one. Consequently there are no more reasons to consider that individual as a customer and by convention his

reactivation probability (PR) is fixed to zero.

Numeric Example

Table 4 illustrates these aspects. Calculations are inspired from Dwyer's example, which has been slightly modified in order to facilitate estimating the reduction of response probability with recency by decision calculus.[4]

A company is forecasting the behaviour of 1000 customers, knowing their purchase probability according to the recency of the last purchase. The marketing margin m is 40$ by transaction and promotion expenses are 4$ by customer. For simplicity reasons these values are kept constant.

Table 4 – The catalogue sales case

The table shows that with a migration model the retention probability is generally low and that companies rely less on the survival probability of a customer than on his reactivation probability in order to boost sales and to increase the customer value.

Comments

The calculation of the purchase probability in column 5 uses the above-mentioned formula in order to illustrate this recurrent retention and reactivation process: t1 = [1*0.3] = 0.300. t2 = [t1*0.3 + t0*(1-0.3)*0.182] = 0.217 etc. This means that an initial transaction t0=1 generates 0.300 transactions after one period and 0.217 transactions after two periods. In the second period the 0.217 transactions come from the two evoked sources: t1*p1 = 0.9 expresses the retention or the survival and t0*(1-p1)*p2 = 1*(1-0.3) *0.182 = 0.127 expresses reactivation of customers that were inactive during the first period. In the fifth period the whole process rests essentially on the reactivation process, as the survival probability diminishes (0.002); at the same time intervenes the censorship of customers who passed the recency limit (R) because they haven't reactivated since 5 periods (their reactivation probability becomes too small).

Formula for a left censored migration process (the starting recency)

The notation tj expresses a customer migration process that starts with a number of manifest customers that is customers that have just ordered or in other words customers for which the recency of the last order is one. The starting number of customers or the number of generating transactions can be noted tr,j. In a customers database important numbers of latent customers or temporarily inactive customers are kept whose recency of the last order is greater than one. The transaction potential of these customers needs to be explicitly considered in certain situation and justifies extending lifetime value calculations to customers' recency greater than one.

In this formula transactional flows with recency greater than one (tr,j, r>1) are expressed as a function of recency one transactional flows. Notations tj and t1,j are equivalent. It is a generalization of the previous formula that accepts left censored migration processes.

Numeric example

A customer with recency two, for example, will have a zero purchase probability at the beginning (column 5 in table 4), in period two his probability to reactivate is 0.18, in period three the purchase probability of 0.145, will be composed of the survival (remain active) probability of 0.055=0.18*0.3 and the probability to reactivate with recency three 0.09=0.92*0.11. After five periods (0 to 4) the purchase probability or the expected number of generated transactions t2,4 becomes 0.062 which is naturally less than t1,4 =0.131 for a customer whose migration process started with the active state (recency 1). The total number of generated transactions T2,4 during the 5 periods becomes 0.5 as compared to T1,4 =1,815. Thus, a customer of recency two can still bring a transaction out of two during the five following periods, while an active customer can bring 2 transactions.

This algebraic formulation of transactional flows in a migration model is equivalent with the matrix formulas suggested by Pfeifer and Carraway and makes a direct link to matrix approach presented in the next section.

Financial flows and customer value calculations

From the number of transactions incurred during each period following the launching of a relationship-marketing program, it is possible to calculate the customer's profitability for the estimated duration of the relationship that is the Lifetime Value. The separation of input (gains) and output (promotion expenses) financial flows in the migration model is needed not only in order to mark the desynchronisation between the two flows (central topic in Berger and Nasr's work) but also in order to reflect the different transactional basis that applies to the two flows.

Customer Value - limits of some formulations

By definition, the migration model considers as a customer every individual having ordered already and not having yet exceeded the pre-established recency limit. This implies that all customers defined in this way are eligible for promotional activities conducted by the company during several periods even if they were not active in the period that precedes these actions. The marketing margin comes only from the active customers while promotional costs are incurred by default with all eligible customers and not only with the active ones as seems to suggest the formula developed by Berger and Nasr [5] . By eliminating the desynchronisation aspect in the timing of gains and expenses, their formula can be reduced to . It reflects a very particular case where promotion costs apply only to the active customers, which is a reduction, compared to the calculations presented by Dwyer, which the authors have followed.

Limited use of calculations adapted from the retention model

For clarity reasons the discount rate is set to zero. The formula becomes then (m-c) or again (m-c)Tr,k . The customer value that results (40$-4 $) *1,815 = 65,34 $, is largely greater than the value in the general case that is presented in the table 4. As might be observed the Berger and Nasr formulation is limited to a very specific situation. Rare are the industries that can count on the spontaneous reactivation of customers without having to make the marketing efforts to stimulate this process.

Calculations adapted to the standard migration model

Companies spend money in order to maintain the retention and reactivation process described by the migration tree. The simplified formula becomes then (m t k - c) or m Tj - c*j. By applying it to the data given in table 4, the customer

value after 5 periods becomes 40 $* 1,815 - 4$*5 = 52,4 $.

Censorship is needed for the economic reasons because, if one continues to engage in promotional actions towards all members of a cohort, costs by active customer increase up to a point where the margin, even if it remains constant, is not enough anymore to cover these costs (see column 9).

With censorship included, the customer value formula is slightly modified to become

[(m tk - c(1-qk))] or m T k -c R+ [c(1-qk)], because when the period j arrives to the recency limit R=4 customers that didn't reactivate are purged (excluded). The customer value becomes 40$*1,815-4$*4-4$*(1-0.475)=54,5 $.

The customer value formula for the standard migration model

The customer value formula suggested here allows for residual value calculations for customers that are inactive at the assessment time (left bound censorship), takes into account the promotional costs that are maintained by default during the lifetime of a customer and integrates customer elimination.

where: = Value of a customer with recency r = expected transactions generated by a customer of recency r in period j; m = net margin on a transaction; c = mailing cost; pl = response probability of a segment of recency l; R = recency limit beyond which a customer is ticked off the data base; a = discount rate; qk = cumulated probability for a customer to be eliminated (purged) after k periods.

This calculation cumulates the present gains produced by transactions that were generated during the projection period and systematically deducts mailing costs. It supposes that mailings are sent to customers only if they are maintained in the database, which is a standard marketing policy in a migration model.

Modelling the customer elimination process

The formula also integrates the modelling of the elimination process for customers that exceed the recency limit. Elimination intervenes when there is a succession of non-response (no-purchase) periods equal to the recency limit.

For an active customer (recency 1) the probability to be eliminated intervenes after R periods and corresponds at each period k to the probability to generate a transaction after k-R+1 periods multiplied with the probability of the non-response succession. In the example in table 4, an active customer, can be eliminated starting with period 4 (the recency limit), with a probability of 0.475, that is the product of the probability to generate a transaction $t_{1,4-4}$ =1 and the probability of the no-answer succession (1-0.3)(1-0.18)(1-0.11)(1-0.06)=0.475.

For an inactive customer (r > 1) the probability to be eliminated intervenes in two stages. The first stage intervenes after R-r+1 periods. For a customer of recency 3 the first elimination intervenes after 2 periods (4-3+1). The second stage begins after R+1 periods (here 4+1=5).

The first stage eliminates customers that don't reactivate anymore with a probability that corresponds to the non-response succession between the recency of the customer and the recency limit. A customer of recency 3 has a probability to be eliminated at the first stage of (1-0.11)(1-0.06)=0.83.

The second stage begins with the reactivation of customers from a cohort with given recency and continues up to the elimination of all customers from this cohort.

The probability to be eliminated at each period k is the product of the probability to generate a transaction after k-R+1 periods that multiplies the probability of the non-response succession. A customer of recency 3 will have in period 5 a probability to be eliminated of 0.0524 = 0.11*0.475. It is the product of the probability to generate a transaction $t_{3,5-4}$ =0.11 and of the probability of the non-response succession 0.475.

Matrix approach

Introduction

The matrix approach suggested by Pfeifer and Carraway (2000) suites both retention and migration models. While for the retention models, the algebraic formulations are simple and straightforward enough, for the migration model the matrix approach represents a flexible and elegant alternative to the algebraic formulas.

Transition probabilities Matrix and its derivatives

Table 5 – Transition probabilities matrix and its transformations

The transition probabilities matrix P in table 5a summarizes the migration process. It expresses transition probabilities, between two periods, from a given recency state either towards recency one (buying customer) or towards a recency state incremented by one (customer who doesn't buy). According to Markov Chain theory the same matrix P to the power 4 (table 5b) indicates transition probabilities after 4 periods. If after one period transition possibilities from a given recency state are limited to two states: recency one and the given recency incremented by one, after two periods possible transitions include three states and after four periods all the five states become accessible. The sum of the P matrixes to powers incremented from 0 to 4 (table 5c) contains the cumulated probabilities during five periods. It should be noticed that the starting period is considered equal to zero and that matrix P to the power zero corresponds to the identity matrix. It can be shown that on the long-term the sum of the probability matrixes P at incremented powers tends on the long term towards the inverse matrix of the difference between the identity matrix and P (table 5d).

Transactional flow

In order to obtain a strict equivalence with the algebraic formulas presented before and summarized in the table 6., we multiply the matrixes in table 5 with the vector noted 11, that has the following shape [1, 0. 0. . .]. In this way the expected number of transactions vector is extracted. It consists of first column elements of the transition probabilities matrix and of its transformations.

Table 6 - The vector of the expected number of transactions in the transition probabilities matrix and its derivatives

The customer value

The value for the anticipated lifetime of a customer can be easily calculated by multiplying the discounted sum of probability matrixes with the rewards vector R. By considering gains and promotional expenses constant, the vector R looks like this: [m-c, -c, -c,..], meaning that customers of recency one bring gains in addition to promotional costs because they buy, while for customers of superior recency there are only costs.

The long-term customer value

By applying the customer value calculation formula to Dwyer's example using a 5 periods time horizon (k=0 to 4) that seems to be this industry's estimated customer lifetime, a customer of recency one has a value of 48,97$, a customer of recency two has a value of 2,52$ and customers with a recency superior to two have negative values. The long-term calculations confirm these results.

Matrix approach : Customer value Optimisation

Introduction

By applying the logic used with retention models it is possible to represent the purchase probability with recency as a function of the marketing effort. To reduce complexity, let's suppose that by controlling the number of mailings sent to customers it is possible to modify the ceiling purchase probability leaving the other parameters unaltered.

Numeric example

In the example the function of purchase probability with recency was 0.3 exp(-0.5*recency-1), where 0.3 is the ceiling (pp) the function reaches when the recency is equal to one and the mailing costs per customer are 4$. If one spends more for mailings this ceiling can grow until a limit of 0.5 and if one spends less it can fall until 0. The ceiling purchase probability expressed as a customer cost function, is pp = 0.3 (1-exp(-0.5 * C)). When the budget is 4$ the ceiling is 0.3 as in Dwyer's example, when the budget is 1$ the ceiling becomes 0.1 and when the budget increases up to 10$ the ceiling approaches the limit of 0.5. Under these circumstances, finding the optimum cost per customer consists of maximising the long-term present value of a recency one customer: max V1 = 11 where R' = [(m-C/pp), - C / pp, - C / pp, - C / pp, 0] and p11 = pp = f(C), and P = f(pp, recency). The Marketing expenses c are C/pp; pp is a function of expenses per customer C and the purchase probability depends on the ceiling pp and the recency.

Optimisation Procedure

By applying this optimisation procedure to the given example it is optimum to spend 3$ per customer and not 4$. The graphical solution is presented in figure 6. Details of the calculations are given in appendix 2.

Figure 6 - Maximization of the long-term customer value for the migration model

It is easy to extend this reasoning to the acquisition of new customers and to find an optimum balance between customer acquisition and retention budgets. In this way the Blattberg and Deighton (1996) optimisation procedure is adapted to customer migration situations.

Optimisation using Markov decision processes

This logic seeks the best trade-off between long-term efforts and gains. For the migration model, other logics with shorter temporal horizons can also be envisaged. Pfeifer and Carraway (2000) present optimisation procedures extracted from the rich literature on Markov decision processes. Birtran and Mondschein (1996) formalize the direct marketing problem (that integrates a particular migration model) as a dynamic programming problem and develop

optimisation heuristics adapted to this operations research problem. These approaches use RFM stratification and vary coverage and intensity of mailing campaigns in order to maximize the value of customers.

Optimising mailing policy without budgetary constraints

It can be shown that without budgetary constraints the optimal policy is to send mailings to all customer segments with positive Lifetime Value. As for monotonously decreasing response probabilities the customer value is also decreasing, when margins and mailing costs are constant, a breakeven response rate can be calculated for which the Lifetime Value is equal to zero. Using this property and the breakeven response rate, it is possible to calculate a profitability index from the response rate of each layer/segment.

Conclusions

Systematisation of calculations

The formulas for computing the economic value of a customer or the customer lifetime value that have been developed in this article are organized in systematic and progressive way according to a double taxonomy: the one of customer relationship behaviour models and the one of calculation methods. The retention/migration model dichotomy is based on strong theoretical foundations in consumer behaviour that have been underlined by the previous studies.

Besides the evoked behavioural characteristics, the comparative analysis of the two categories of customer relationship models reveals their economic substrata. In a migration model the retention probability is usually relatively low and companies rely less on the customer's survival probability than on his reactivation probability in order to increase sales and customer value.

Progressive modelling

The two calculation methods, algebraic and matrix based, are stepwise and progressively applied to both relationship behaviour models. On that occasion several stages are distinguished 1) a physical, quantitative level of transaction

flows that are stochastically treated, 2) a monetary value level of financial flows, the temporal effects of which are studied, and 3) a decisions level the one of optimisation calculations. Calculations become in this way easier to compare and their formalisation in order to solve problems of increasing complexity becomes easier.

New algebraic formulas

The algebraic formulas developed for the migration model represent an alternative to the matrix formulations. They make it possible to approach in an explicit manner the desynchronisation of financial flows and the censorship of migration processes on recency. Calculations to estimate eliminations from the customer list formalizes right censorship in a migration process; computing the probabilities to generate transactions for inactive customers formalizes left censorship. These aspects are only implicitly treated in the matrix approach of Pfeifer and Carrway (2000) and are absent from the algebraic approach suggested by Berger and Nasr (1998).

A matrix optimisation procedure

The optimisation procedure developed for retention models as well as the Blattberg and Deighton procedure that optimally balances marketing efforts between customer acquisition and retention have also been adapted to the migration model using the properties of long-term transition matrixes.

Application conditions

As shown by Mulhern (1999) customer LTV models cannot be applied to all situations. They require customers to have some persistent relations with enterprises and that financial flows (gains and expenses) can be forecasted with a certain precision at an individual level. When these conditions are not satisfied, the historic analysis of profitability can replace LTV calculations. Yet, under the combined and interdependent impulses from information technology progress and from the adoption of a customer centred marketing, situations for which models of LTV are applicable become dominant. This incites pursuing the customer evaluation and dynamic management models' systematisation and unification efforts.

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Appendix 2: Matrix calculations for customer long term value optimisation

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